A Gateway to All Post Graduate Courses

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**Subject:** Physics

Production of Courseware

-Content for Post Graduate Courses

**Paper No.** : Quantum Mechanics-I

**Module** : Some Earlier Attempts Towards Quantum Mechanical Concepts-I

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| Description of Module | |
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| **Subject Name** | Physics |
| **Paper Name** | Quantum Mechanics-I |
| **Module Name/Title** | Some Earlier Attempts Towards Quantum Mechanical Concepts- I |
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**Contents of this Unit**



**Learning Outcomes**
After studying this module, you shall be able to

- Learn a few basic processes which show how interaction of electromagnetic waves with matter is visualized to occur by means of elementary processes in which the radiation appears to be composed of discrete quanta of energy called photons
- Know that the energy E and momentum p of a photon are related to the wave characteristics like the frequency ν and the wave vector $\vec{k}$ by the relations $E = h\nu$ *and* $\vec{p} = \hbar\vec{k}$ , where h is Planck's constant and $\hbar = h/2\pi$ .
- Learn the concept of Wave- Particle duality by analyzing Young's double-slit interference experiment.
- Know that (i) light behaves simultaneously as a wave and as a stream of independent photons, (ii) the possibility of explaining interference fringes due to interaction of photons is ruled out, (iii) the wave nature of photons makes it impossible to detect through which slit a photon has passed to produce the interference fringes; it only enables us to determine the probability of striking the screen at a point.

## 1.INTRODUCTION

The subject of quantum mechanics provides a mathematical description to understand the phenomena which occur on a microscopic ( atomic or subatomic) scale. In fact, the subject grew as a result of culmination of new and path-breaking ideas introduced to account for the observed phenomena which could not be explained on the basis of classical concepts. This module and the one following it, present a review of some of the basic concepts and ideas which lay the groundwork for a systematic development of the subject.

## 2.ELECTROMAGNETIC WAVES AND PHOTONS

### 2.1 Light Quanta and the Planck-Einstein Relation :

One of the earliest phenomena which electromagnetic theory was unable to explain was the nature of the distribution of energy in the spectrum of radiation from a black body. As you know, a black body, by definition, is one which absorbs all the radiation it receives and does not reflect any ('black'!). Such objects are perfect absorbers as well as emitters. A small opening of a hollow enclosure ( as for instance, an oven) can be the best practical realization of a black body. Such a cavity with a small aperture through which radiation from outside may be admitted contains radiations emitted by the walls of the enclosure. The spectrum of the radiation is characterized by a function $E(\nu)$ , where $E(\nu)\, d\nu$ represents energy (per unit volume of the cavity) of the radiation with frequencies between $\nu$ and $\nu+d\nu$..

Experimental observations, as depicted in Fig.(1.1), show that the spectral distribution of black body radiation, which depends only on the temperature, plotted as a function of wavelength $\lambda$ for different absolute temperatures falls off after reaching a maximum as wavelength is increased.

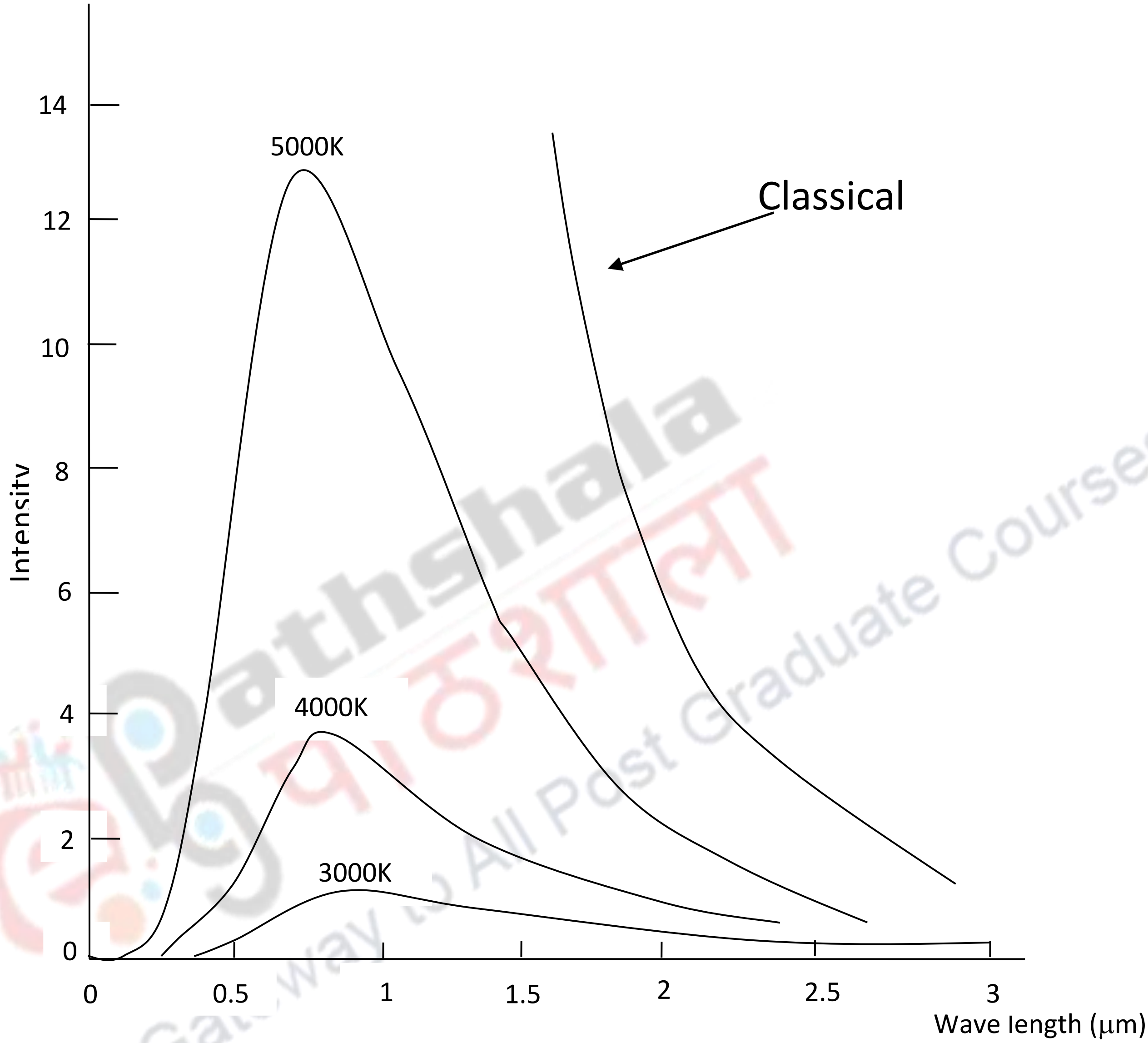


Fig. 1.1

**Fig. 1.1** *Spectral distribution of black body radiation. Plot of energy density distribution at different absolute temperatures as a function of wavelength.*

Earlier attempts based on classical ideas were found to be inadequate to explain the observed frequency distribution of the black body. Thus, for instance, Wien suggested semi-empirical theory which agreed only in the short wavelength limit, while Raleigh and Jeans deduced, from classical reasoning, a law which agreed at long wavelengths limit but was in complete disagreement for short wavelengths. The basis of their argument was that according to classical electromagnetic theory, the energy density distribution, E(ν), of a black body radiation can be expressed as

$$E(\nu) = \frac{8\pi\nu^2}{c^3} u(\nu) ,$$

where $8\pi\nu^2 / c^3$ is the number of e.m oscillators per unit volume at frequency ν in the range dν in equilibriorum with the radiation in a black body cavity and u(ν) represents the average energy of the oscillators. From the 'Law of Equipartition of Energy', the mean energy of the oscillators with frequency ν is

u(ν)=k T ,

where k is the Boltzmann's constant, and T is the absolute temperature. Thus we get

$$E(\nu)\,d\nu = \frac{8\pi\nu^2}{c^3} kT\,d\nu$$

*or*

$$E(\lambda)d\lambda = \frac{8\pi}{\lambda^4} kTd\lambda$$

This is known as Rayleigh-Jean's law. According to this law the energy radiated in a given wavelength range dλ increases indefinitely as λ becomes smaller and smaller. This is clearly in complete disagreement with the observed spectral distribution.

To explain the observed features, it led Planck to suggest the **hypothesis of the quantization of energy (1900)**:- For an electromagnetic wave of frequency ν, the only possible energies are integral multiples of the quantum of energy, **ε= hν**. Planck did not introduce the concept of photons. He, however, suggested the emission and absorption of radiation by matter takes place in discrete quanta of energy.

Planck also assumed that a black body is composed of oscillators in equilibrium with radiation field. The number of oscillators with energy in equilibrium with temperature T can be obtained from the classical Boltzmann expression and is given by:

$$N_\nu = \exp(-\varepsilon_n / kT) / \sum_{n=0}^{\infty} \exp(-\varepsilon_n / kT), \tag{1.1}$$

where $N_\nu$ is the number of oscillators of frequency $\nu$. The mean energy per oscillator of frequency $\nu$ is

$$U(\nu) = \sum_{n=0}^{\infty} \varepsilon_n \exp(-\varepsilon_n / kT) / \sum_{n=0}^{\infty} \exp(-\varepsilon_n / kT), \tag{1.2}$$

where $\varepsilon_n = n$. Now

$$\sum_{n=0}^{\infty} \exp(-\varepsilon_n / kT) = 1 + \exp(-h\nu / kT) + \exp(-2h\nu / kT) + \ldots\ldots.$$

$$= [1 - \exp(-h\nu / kT)]^{-1}$$

Similarly, it can be shown that

$$\sum_{n=0}^{\infty} \varepsilon_n \exp(-\varepsilon_n / kT) = \frac{h\nu \exp(-h\nu / kT)}{[1 - \exp(-h\nu / kT)]^2}$$

Thus $$U(\nu) = \frac{h\nu \exp(-h\nu / kT)}{[1 - \exp(-h\nu / kT)]} = \frac{h\nu}{[\exp(h\nu / kT) - 1]} \qquad (1.3)$$

The number of oscillators per unit volume is easily shown as $n(\nu) = \dfrac{8\pi\nu^2}{c^3}$. We thus get Planck's expression for the density of radiation,

$$E(\nu) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{[\exp(h\nu / kT) - 1]}, \qquad (1.4)$$

which agrees very closely with the observed radiation distribution.

In the long wavelength (or low frequency) limit, we can write

$$e^{h\nu / kT} \approx 1 + \frac{h\nu}{kT}.$$

Substituting it in Eq.(1.4), Planck's expression reduces to that obtained by Rayleigh-Jeans.
The total radiation density can be obtained from Eq.(1.4) by integrating over all frequencies, viz.,

$$E = \int_0^{\infty} E(\nu)\, d\nu = \frac{8}{15} \frac{\pi^5 k^4}{c^3 h^3} T^4,$$

showing that the energy density is proportional to the 4th power of the absolute temperature. This is known as **Stefan's Law** – suggested first by Stefan.

**2.2 Photoelectric Effect**:
About 5 years later, in 1905, Einstein showed how the phenomenon of photoelectric effect, hitherto unexplained by classical theory, could be described in a simple way by considering a monochromatic beam of light not as waves but as bundles of light quanta of energy, h ν. When a beam of light of frequency ν is incident on a metal, it takes a certain amount of minimum energy, called the work function W to remove the electron from the metal. Thus, if      is defined as the threshold frequency, W= h     . When the incident frequency ν is greater than     , the ejected electron has kinetic energy KE, given by

KE= h ν – h (1.5)

which is found to vary linearly with the frequency of incident radiation but is independent of its intensity. *In contrast*, *a*ccording to the wave picture of light, the ‘free electrons’ at the surface of the metal over which the beam of radiation falls absorb the radiant energy continuously. The greater the intensity of radiation, the greater should be the energy absorbed by the electron. Therefore, the maximum kinetic energy of the photoelectrons should, according to wave theory, increase with increase in intensity. This is in contradiction with the experimental observation.

( ii) It is experimentally observed that for a frequency of incident radiation less than the cut off frequency, no photoelectron emission is possible, even if the intensity is large. Whereas in the wave picture, no matter what the frequency of radiation is, a sufficiently intense beam of radiation should be able to impart enough energy so as to exceed the minimum energy needed to escape the electrons; a threshold frequency should, therefore, not exist.

(iii) Experimentally, the photoelectric emission is an instantaneous process without any apparent time lag (less than or of the order of $10$ ). In wave picture, the absorption of energy takes place continuously. The energy absorbed per electron per unit time turns out to be small. Explicit calculations show that it can take a sufficient long time for a single electron to pick up sufficient energy to overcome the work function in order to come out of the metal.

**2.3The Compton Effect**:

About 20 years later, i.e., in 1924, Arthur Compton discovered that when hard X-rays (of shorter wavelength) are scattered by atoms of an element of low atomic number (such as graphite), the scattered radiation contains not only the original wavelength but also softer X-rays of longer wavelength. Compton was able to explain this phenomenon by assuming that X-rays consist of a collection of photons, each characterized by energy, E, and momentum, p. Assuming that X-rays of wavelength $\lambda$ consist of a stream of photons of energy E $= h\nu = hc/\lambda$, Compton argued that when one of these quanta hits a free or loosely bound electron, it would recoil. As a result it would have an energy $E'$ after the collision and therefore the corresponding wavelength $\lambda' \rangle \lambda$. Based on this picture, quantitative calculations can be made, using the laws of conservation of energy and momentum, to estimate the increase in the wavelength of the scattered photon.

Consider an incident photon of energy and momentum , which collides with an electron and is then scattered as shown in the figure ( Fig.1.2 ). The figure shows the scattered photon of energy $E_1$ *and momentum* $p_1 = E$ making an angle and the recoiling electron of momentum making angle ϕ with the direction of the incident photon.

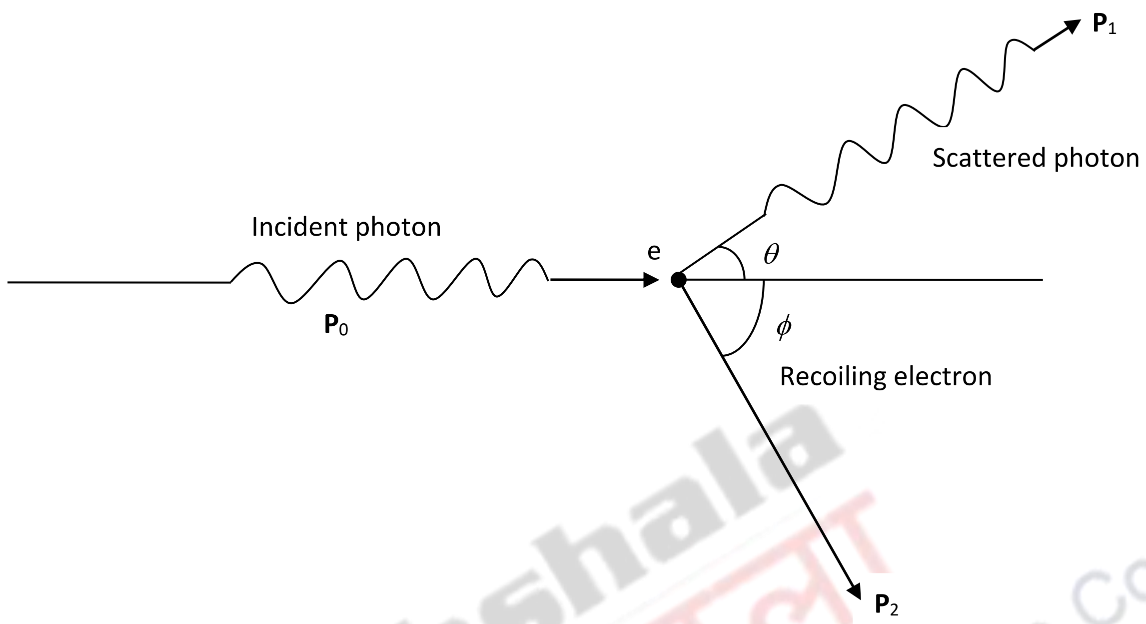


Fig.

**Fig. 1.2** *Incident photon of momentum on a free electron at rest. After the collision, the electron recoils with momentum , while the scattered photon has momentum*

For materials of low atomic number, the energy of the hard X-rays may be very large compared to the electron binding energy. We may therefore assume the energy of the electron before collision to be the rest energy, i.e., $W_0 = mc^2$, where m is the rest mass of the electron. However, since the recoiling electron from the collision may have velocity comparable to c, it would just be appropriate to use the relativistic relation for the velocity v of the recoiling electron. Using the law of conservation of energy during collision, we have

$$E_0 = E_1 + \qquad (1.6)$$

where T is the kinetic energy of the recoiling electron given by

$$\begin{aligned} T &= (mc^2 + c^2 p_2^2)^{1/2} - mc^2 \\ &= c(p_0 - p_1) \end{aligned} \qquad (1.7)$$

Given that the momenta of the photons before and after scattering are

$$p_0 = \frac{E_0}{c} = \frac{h\nu}{c} = \frac{h}{\lambda_0}, \qquad \textit{and} \qquad p_1 = \frac{E_1}{c} = \frac{h\nu_1}{c} = \frac{h}{\lambda_1} \quad . \qquad (1.8)$$

and applying the law of conservation of momentum, we have

$$p_0 = p_1 \cos\theta + p_2 \cos\phi$$
$$0 = p_1 \sin\theta - p_2 \sin\phi \quad (1.9)$$

On solving the equations (1.6) to (1.9), we can show that the increase in wavelength is given by

$$\Delta\lambda = \lambda_1 - \lambda_0 = \frac{h}{mc}(1-\cos\theta). \quad (1.10)$$

To sum up, the results from the studies of these phenomena bring about the fact that the interaction of an electromagnetic wave with matter occurs by means of elementary indivisible processes in which the radiation appears to be composed of particles – **the photons**. The energy, E, and its momentum, p, are related to the wave characteristics – the frequency, ν, and the wave vector (where the magnitude of the wave vector , k= $2\pi/\lambda$) by the relations:

$$E = h\nu = \hbar\omega \qquad and \qquad \vec{p} = \hbar\vec{k} \quad , \quad (1.11)$$

where $\hbar = h/2\pi$ and Planck's constant h=6.63 $\times 10^{-34} J$ .

## 3.Wave- particle Duality:

You have already studied that the phenomena such as diffraction, interference and polarization could only be explained by the wave nature of light and not through the particle picture. We shall now show by analyzing the well-known Young's double slit experiment that light, in fact, exhibits a **dual nature – both the wave aspect and the particle aspect**.

### 3.1 Young's Double- Slit Experiment:

The figure given below shows a schematic arrangement of Young's double slit experiment, where monochromatic light emitted by the source S is allowed to pass through two slits, $S_1$ *and* to illuminate the screen D ( which may, for example, be a photographic plate). If we block slit , we would observe on the screen a diffraction pattern having intensity distribution $I_1$ due to slit . Similarly, by blocking the slit , we would get on the screen a diffraction pattern of intensity $I_2$ due to slit . When both the slits are open, we observe a pattern of interference fringes on the screen. Note that the intensity distribution, I(x), is not the sum of the intensities, $I_1$ and $I_2$ , produced by the two slits separately, i.e., I(x) $\neq I_1(x) + I_2($

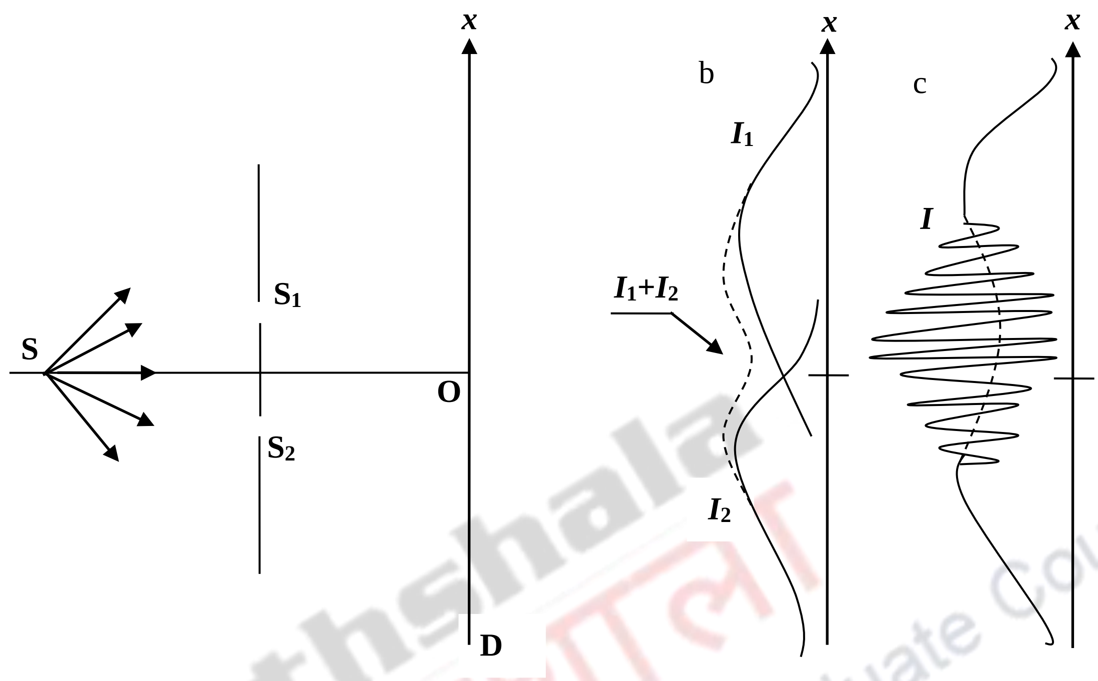


**Fig. 1.3**

**Fig. 1.3** *Schematic diagram of Young's double- slit interference experiment. Each of the two slits, and produces a diffraction pattern ( depicting the intensity distribution shown in (b)) on the screen D . When the two slits are open simultaneously, the intensity I observed on the screen shows an interference pattern ( as depicted in (c) ) on the screen.*

You may recall from your earlier studies that electromagnetic wave theory provides a natural interpretation of the formation of interference fringes. The intensity of light at a point on the screen D is proportional to the square of the amplitude of the electric field at this point. If the two slits, $S_1$ *and* produce respectively the electric fields, $\mathrm{E}_1(x)$ *and* $\mathrm{E}_2(x)$, at a point x on the screen, the resultant electric field at that point is

$$\mathrm{E}(x) = \mathrm{E}_1(x) + \mathrm{E}_2(x)\,, \tag{1.12}$$

giving the total intensity, $$I(x) \propto |\mathrm{E}(x)|^2 = |\mathrm{E}_1(x) + \mathrm{E}_2(x)|^2\ , \tag{1.13}$$

where $\mathrm{E}_1(x)$ *and* $\mathrm{E}_2(x)$, being complex, the right hand side in Eqn.(1.13) represents modulus squared. The important point to note from this equation is that the intensity I(x) differs from $I_1$ and $I_2$ by an interference term which depends on the phase difference between $\mathrm{E}_1(x)$ *and* $\mathrm{E}_2(x)$. It is the interference term which accounts for the formation of interference fringes on the screen. The wave theory also predicts that by reducing the intensity of the source S, the intensity of the fringes will get reduced but the fringes will not disappear.

If we attempt to explain the observed interference pattern by assuming interaction between the photons which pass through the slit and those which pass through the slit , it would lead to the following prediction:

Let the intensity of the source S (i.e., the number of photons emitted per second) be reduced until the photons strike the screen practically one by one thereby reducing the possibility of interaction between the photons and making it eventually vanish altogether. In such a scenario, the interference fringes should vanish. But what we observe is that by covering the screen with a photographic plate and increasing the exposure time so as to collect a large number of photons on the plate, we would observe that the fringes have not disappeared. Thus the possibility of explaining the occurrence of interference fringes arising due to interaction between photons, i.e., on the basis of pure particle picture of light, can just be ruled out.

If, on the other hand, we expose the photographic plate for a time so short that it receives only a few photons, we would observe that each photon has only a localized effect and would not result in producing an interference ( howsoever weak) pattern. In actual practice, what happens is that as more and more photons strike the plate, their individual impacts seem to be distributed in a random manner. It is only when a large number of them have reached the screen, does the distribution of impacts starts appearing. The density of impacts at each point on the screen results in the intensity distribution corresponding to the interference pattern.

In this experiment, since photon-photon interactions are excluded, suppose we make an attempt to find out through which slit each photon passed before reaching the screen. To obtain this information, let us imagine placing detectors behind the slits $S_1$ *and* $S_2$. We will the find that if the photons arrive one by one, each one that passes through a particular slit will be recorded through a signal either by the detector placed behind $S_1$ or one behind $S_2$. Clearly the photons detected in this way are being absorbed by the respective detectors and do not reach the screen. Let us, for instance, remove the detector which blocks only $S_1$. Then the one which is behind $S_2$ would tell us that out of a large number of photons, about half pass through $S_2$. The others which reach the screen passing through $S_1$ do not develop the interference pattern, since $S_2$ is blocked, but produce, on the other hand, only a diffraction pattern.

In this particle picture of photons, as photon-photon interactions are excluded, each photon must be considered separately. The puzzling question then is this: why the phenomenon should change drastically depending on whether only one or both the slits are open? In other words, for a photon passing through one of the slits why should the fact that the other is open or closed make such a fundamental difference?

### 3.2 Quantum –mechanical Viewpoint:

In the question just raised, the implicit assumption is that the photon actually goes through a particular one of the two slits. From the point of view of classical theory, this assumption is natural since classically each particle or a photon has definite and deterministic position at each instant of time. **At the quantum mechanical or a microscopic level, this picture is to be discarded**. For example, an essential characteristic of this new domain appeared when we placed detectors behind the slits by trying to detect through which slit the photons crossed the slits. In fact, a detailed analysis shows that it is impossible to observe the interference pattern and to know at the same time through which slit each photon had passed.

From the above experiment we have also seen that as the photons reach the screen one by one, their impacts gradually build up the interference pattern ; that is to say, for a particular photon, we can not anticipate where it will strike the screen. We can only say that the probability of its striking the screen at a point x is proportional to $|\mathrm{E}(x)|^2$. Here too, we are discarding another classical idea, viz., 'the motion of a particle at any time can be uniquely predicted from its motion at an earlier time'. We can thus summarize the concept of **wave-particle duality,** as:

(a) Light behaves simultaneously as a wave and as a stream of independent photons . The wave nature, which appears to be an inherent property of photon, enables us to calculate the probability of the manifestation of a photon.

(b) The probability amplitude of a photon appearing at a time t at the point **r** can bedescribed by the electromagnetic wave represented by E(**r,** t) and the corresponding probability is proportional to $|\mathrm{E}(\vec{r},t)|^2$.

## 4.SUMMARY

In this module, you study

- How the phenomena of black body radiation, photoelectric effect and Compton scattering bring about the fact that the interaction of an electromagnetic wave with matter occurs by means of elementary indivisible processes in which the radiation appears to be composed of particles – **the photons**.
- The energy, E, and its momentum, p, are related to the wave characteristics – the frequency, ν, and the wave vector (where the magnitude of the wave vector , k= $2\pi/\lambda$ ) by the relations:

$$E = h\nu = \hbar\omega \qquad and \qquad \vec{p} = \hbar\vec{k} \quad ,$$

- The concept of **wave-particle duality** through Young's double slit experiment to demonstrate that

(a) Light behaves simultaneously as a wave and as a stream of independent photons . The wave nature, which appears to be an inherent property of photon, enables us to calculate the probability of the manifestation of a photon.

(b) The probability amplitude of a photon appearing at a time t at the point **r** can be described by the electromagnetic wave represented by E(**r,** t) and the corresponding probability is proportional to $|\mathrm{E}(\vec{r},t)|^2$.