Wave Particle Duality

Light can exhibit both kind of nature of waves and particles so the light shows wave-particle dual nature.

In some cases like interference, diffraction and polarization it behaves as wave while in other cases like photoelectric and compton effect it behaves as particles (photon).

De Broglie Waves

Not only the light but every materialistic particle such as electron, proton or even the heavier object exhibits wave-particle dual nature.

De-Broglie proposed that a moving particle, whatever its nature, has waves associated with it. These waves are called "**matter waves**".

Energy of a photon is

$$E = h\nu$$

For a particle, say photon of mass, m

$$E = mc^2$$

$$mc^2 = h\nu$$

$$mc^2 = \frac{hc}{\lambda}$$

$$\lambda = \frac{h}{mc}$$

Suppose a particle of mass, m is moving with velocity, v then the wavelength associated with it can be given by

$$\lambda = \frac{h}{mv} \quad \text{or} \quad \lambda = \frac{h}{p}$$

(i) If $v = 0 \Rightarrow \lambda = \infty$ means that waves are associated with **moving** material particles only.

(ii) De-Broglie wave does not depend on whether the moving particle is charged or uncharged. It means matter waves are not electromagnetic in nature.

Wave Velocity or Phase Velocity

When a monochromatic wave travels through a medium, its velocity of advancement in the medium is called the wave velocity or phase velocity (V_p).

$$V_p = \frac{\omega}{k}$$

where $\omega = 2\pi\nu$ is the angular frequency

and $k = \frac{2\pi}{\lambda}$ is the wave number.

Group Velocity

In practice, we came across pulses rather than monochromatic waves. A pulse consists of a number of waves differing slightly from one another in frequency.

The observed velocity is, however, the velocity with which the maximum amplitude of the group advances in a medium.

So, the group velocity is the velocity with which the energy in the group is transmitted (V_g).

The individual waves travel "inside" the group with their phase velocities.

$$V_g = \frac{d\omega}{dk}$$

Relation between Phase and Group Velocity

$$V_g = \frac{d\omega}{dk} \quad = \frac{d}{dk}(kV_p)$$

$$V_g = V_p + k\frac{dV_p}{dk}$$

$$V_g = V_p + \frac{2\pi}{\lambda}\frac{dV_p}{d(2\pi/\lambda)}$$

$$V_g = V_p + \frac{1}{\lambda}\frac{dV_p}{d(1/\lambda)}$$

$$V_g = V_p + \frac{1}{\lambda}\frac{dV_p}{\left(-\frac{1}{\lambda^2}d\lambda\right)}$$

$$V_g = V_p - \lambda\frac{dV_p}{d\lambda}$$

In a Dispersive medium V_p depends on frequency

i.e. $\frac{\omega}{k} \neq$ constant

So, $\lambda\frac{dV_p}{d\lambda}$ is positive generally (not always).

$\Rightarrow V_g < V_p$ generally

$$V_g = V_p - \lambda\frac{dV_p}{d\lambda}$$

In a non-dispersive medium (such as empty space)

$\frac{\omega}{k} =$ constant $= V_p$

$$\Rightarrow \frac{dV_p}{d\lambda} = 0$$

$$\Rightarrow V_g = V_p$$

Phase Velocity of De-Broglie's waves

According to De-Broglie's hypothesis of matter waves

$$\lambda = \frac{h}{mv}$$

wave number $$k = \frac{2\pi}{\lambda} = \frac{2\pi mv}{h} \qquad \text{(i)}$$

If a particle has energy E, then corresponding wave will have frequency

$$\nu = \frac{E}{h}$$

then angular frequency will be $\omega = 2\pi\nu = \frac{2\pi E}{h}$

$$\omega = \frac{2\pi mc^2}{h} \qquad \text{(ii)}$$

Dividing (ii) by (i)

$$\frac{\omega}{k} = \frac{2\pi mc^2}{h} \times \frac{h}{2\pi mv}$$

$$V_p = \frac{c^2}{v}$$

But v is always < c (velocity of light)

(i) Velocity of De-Broglie's waves $V_p > c$ (not acceptable)

(ii) De-Broglie's waves (V_p) will move faster than the particle velocity (v) and hence the waves would left the particle behind.

Group Velocity of De-Broglie's waves

The discrepancy is resolved by postulating that **a moving particle is associated with a "wave packet" or "wave group", rather than a single wave-train.**

A wave group having wavelength λ is composed of a number of component waves with slightly different wavelengths in the neighborhood of λ.

Suppose a particle of rest mass m_o moving with velocity v then associated matter wave will have

$$\omega = \frac{2\pi mc^2}{h} \quad \text{and} \quad k = \frac{2\pi mv}{h} \quad \text{where} \quad m = \frac{m_o}{\sqrt{1 - v^2/c^2}}$$

$$\omega = \frac{2\pi m_o c^2}{h\sqrt{1 - v^2/c^2}} \quad \text{and} \quad k = \frac{2\pi m_o v}{h\sqrt{1 - v^2/c^2}}$$

On differentiating w.r.t. velocity, v

$$\frac{d\omega}{dv} = \frac{2\pi m_o v}{h\left(1 - v^2/c^2\right)^{3/2}} \qquad \text{(i)}$$

$$\frac{dk}{dv} = \frac{2\pi m_o}{h\left(1 - v^2/c^2\right)^{3/2}} \qquad \text{(ii)}$$

Dividing (i) by (ii)

$$\frac{d\omega}{dv}\cdot\frac{dv}{dk}=\frac{2\pi m_o v}{2\pi m_o}$$

$$\frac{d\omega}{dk}=v=V_g$$

Wave group associated with a moving particle also moves with the velocity of the particle.

Moving particle $\equiv$ wave packet or wave group