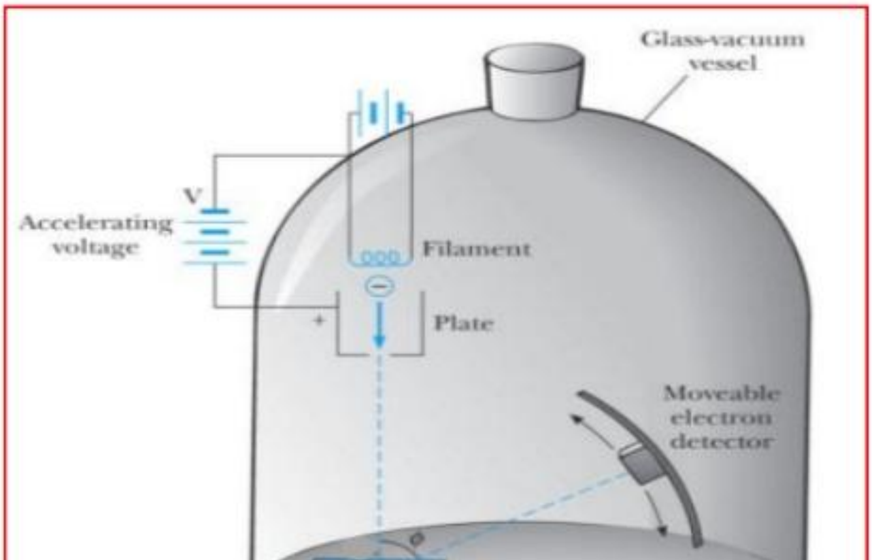
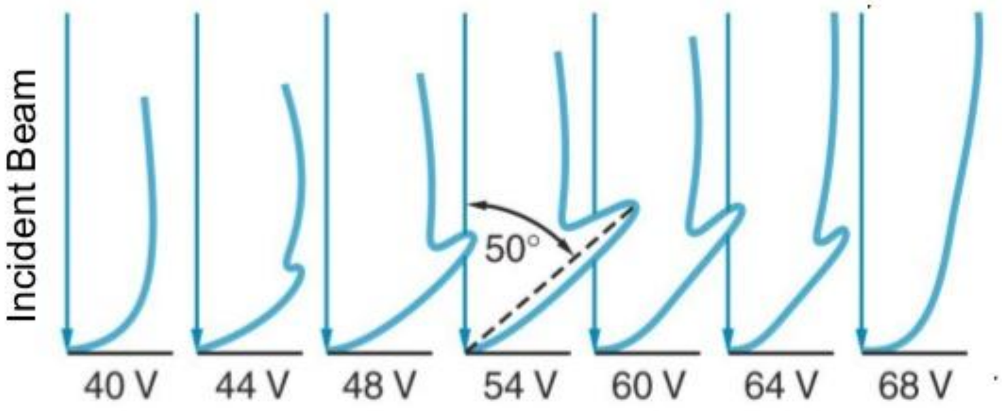
Davisson & Germer experiment of electron diffraction

- If particles have a wave nature, then under appropriate conditions, they should exhibit diffraction
- Davisson & Germer measured the wavelength of electrons
- This provided experimental confirmation of the matter waves proposed by de Broglie

Davisson and Germer Experiment





Current vs accelerating voltage has a maximum (a bump or kink noticed in the graph), i.e. the highest number of electrons is scattered in a specific direction.

The bump becomes most prominent for 54 V at $\varphi \sim 50°$

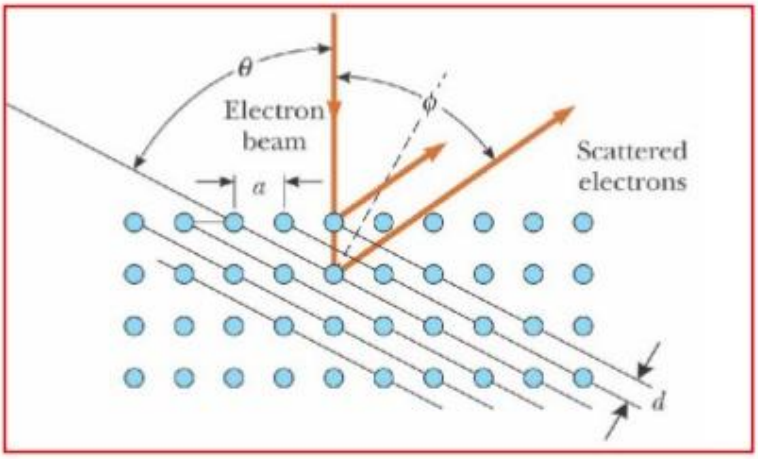
According to de Broglie, the wavelength associated with an electron accelerated through V volts is

$$\lambda = \frac{12.28}{\sqrt{V}} \overset{o}{A}$$

Hence the wavelength for 54 V electron

$$\lambda = \frac{12.28}{\sqrt{54}} = 1.67 \overset{o}{A}$$

From X-ray analysis we know that the nickel crystal acts as a plane diffraction grating with grating space d = 0.91 Å



Here the diffraction angle, $\varphi \sim 50°$

The angle of incidence relative to the family of Bragg's plane

$$\theta = \left(\frac{180^o - 50^o}{2} \right) = 65^o$$

From the Bragg's equation

$$\lambda = 2d\sin\theta$$

$$\lambda = 2\times(0.91\,\overset{o}{A})\times\sin 65^{o} = 1.65\,\overset{o}{A}$$

which is equivalent to the λ calculated by de-Broglie's hypothesis.

It confirms the wavelike nature of electrons

Heisenberg Uncertainty Principle

It states that only one of the "position" or "momentum" can be measured accurately at a single moment within the instrumental limit.

or

It is impossible to measure both the position and momentum simultaneously with unlimited accuracy.

$\Delta x \rightarrow$ uncertainty in position

$\Delta p_x \rightarrow$ uncertainty in momentum

then $\Delta x \Delta p_x \geq \frac{\hbar}{2}$ $\quad \therefore \hbar = \frac{h}{2\pi}$

The product of Δx & Δp_x of an object is greater than or equal to $\frac{\hbar}{2}$

If Δx is measured accurately i.e. $\Delta x \rightarrow 0 \quad \Rightarrow \Delta p_x \rightarrow \infty$

The principle applies to all canonically conjugate pairs of quantities in which measurement of one quantity affects the capacity to measure the other.

Like, energy E and time t.

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

and angular momentum L and angular position θ

$$\Delta L \Delta \theta \geq \frac{\hbar}{2}$$

Determination of the position of a particle by a microscope

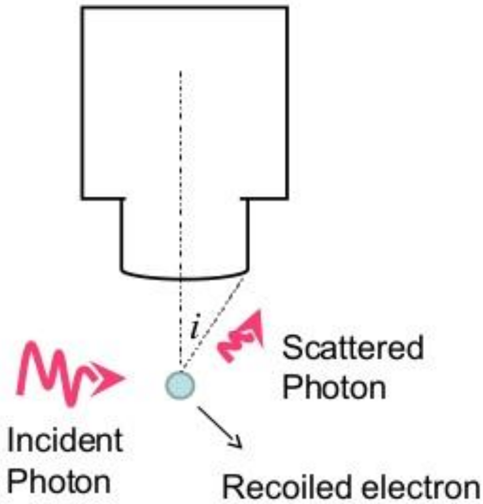
Suppose we want to determine accurately the position and momentum of an electron along x-axis using an ideal microscope free from all mechanical and optical defects.

The limit of resolution of the microscope is

$$\Delta x = \frac{\lambda}{2\sin i}$$

here *i* is semi-vertex angle of the cone of rays entering the objective lens of the microscope.

Δx is the order of uncertainty in the x-component of the position of the electron.



We can't measure the momentum of the electron prior to illumination. So there is uncertainty in the measurement of momentum of the electron.

The scattered photon can enter the microscope anywhere between the angular range +*i* to –*i*.

The momentum of the scattered photon is (according to de-Broglie)

$$p = \frac{h}{\lambda}$$

Its x-component can be given as

$$\Delta p_x = \frac{2h}{\lambda}\sin i$$

The x-component of the momentum of the recoiling electron has the same uncertainty, Δp_x (conservation of momentum)

The product of the uncertainties in the x-components of position and momentum for the electron is

$$\Delta x.\Delta p_x = \frac{\lambda}{2\sin i} \times \frac{2h}{\lambda}\sin i$$

$$\Delta x.\Delta p_x = h > \frac{\hbar}{2}$$

This is in agreement with the uncertainty relation.

Applications of Heisenberg Uncertainty Principle

(i) Non-existence of electron in nucleus

Order of radius of an atom ~ 5 x10^{-15} m

If electron exist in the nucleus then $(\Delta x)_{\max} = 5\times 10^{-15}\, m$

$$\Delta x \Delta p_x \geq \frac{\hbar}{2}$$

$$(\Delta x)_{\max}(\Delta p_x)_{\min} = \frac{\hbar}{2}$$

$$(\Delta p_x)_{\min} = \frac{\hbar}{2\Delta x} = 1.1\times 10^{-20}\, kg.m.s^{-1}$$

then $E = pc = 20\, MeV$ $\therefore$ relativistic

Thus the kinetic energy of an electron must be greater than 20 MeV to be a part of nucleus

Experiments show that the electrons emitted by certain unstable nuclei don't have energy greater than 3-4 MeV.

Thus we can conclude that the electrons cannot be present within nuclei.

Concept of Bohr Orbit violates Uncertainty Principle

$$\Delta x.\Delta p \geq \frac{\hbar}{2}$$

But $$E = \frac{p^2}{2m}$$

$$\Delta E = \frac{p\Delta p}{m} \quad = \frac{mv\Delta p}{m} \quad = \frac{\Delta x}{\Delta t}\Delta p$$

$$\Delta E.\Delta t = \Delta x.\Delta p$$

$$\Delta E.\Delta t \geq \frac{\hbar}{2}$$

According to the concept of Bohr orbit, energy of an electron in a orbit is constant i.e. ΔE = 0.

$$\Delta E.\Delta t \geq \frac{\hbar}{2}$$

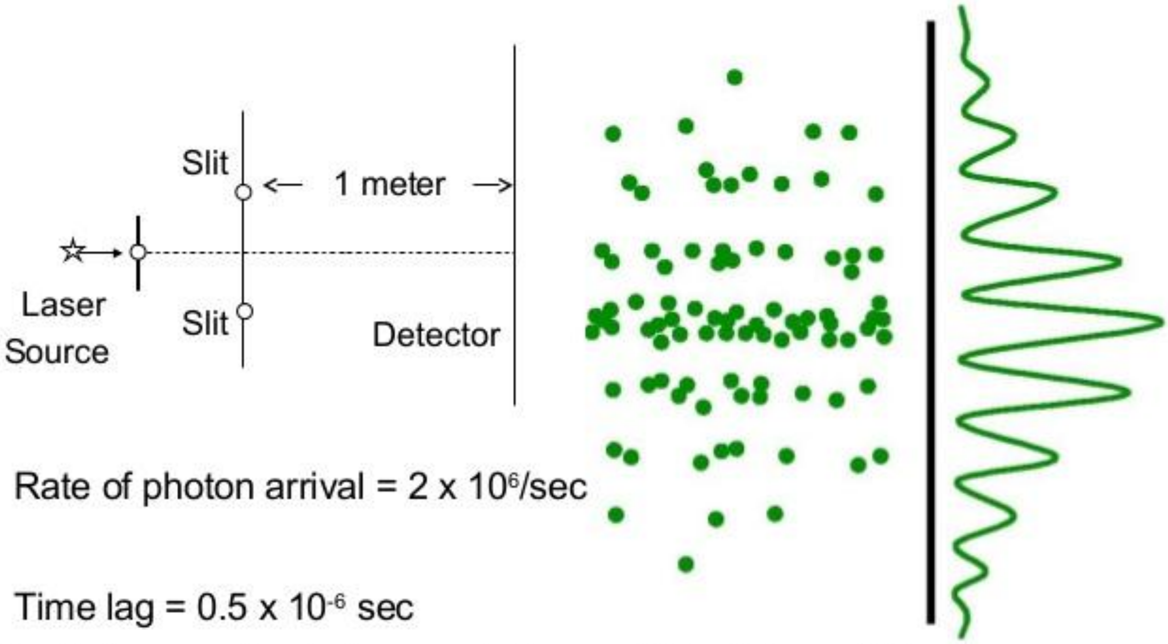
$$\Rightarrow \Delta t \rightarrow \infty$$

All energy states of the atom must have an infinite life-time.

But the excited states of the atom have life–time ~ 10^{-8} sec.

The finite life-time Δt gives a finite width (uncertainty) to the energy levels.

Two-slit Interference Experiment



Rate of photon arrival = 2×10^6/sec

Time lag = 0.5×10^{-6} sec

Spatial separation between photons = 0.5×10^{-6} c = 150 m

- Taylor's experiment (1908): double slit experiment with very dim light: interference pattern emerged after waiting for few weeks
- interference cannot be due to interaction between photons, i.e. cannot be outcome of destructive or constructive combination of photons

  ⇒ interference pattern is due to some inherent property of each photon - it "interferes with itself" while passing from source to screen
- photons don't "split" –

  light detectors always show signals of same intensity
- slits open alternatingly: get two overlapping single-slit diffraction patterns – no two-slit interference
- add detector to determine through which slit photon goes:

  ⇒ no interference
- interference pattern only appears when experiment provides no means of determining through which slit photon passes

Double slit expt. -- wave vs quantum

wave theory

- pattern of fringes:
  - Intensity bands due to variations in square of amplitude, A^2, of resultant wave on each point on screen
- role of the slits:
  - to provide two coherent sources of the secondary waves that interfere on the screen

quantum theory

- pattern of fringes:
  - Intensity bands due to variations in probability, P, of a photon striking points on screen
- role of the slits:
  - to present two potential routes by which photon can pass from source to screen