Quantum Mechanics

It is a generalization of Classical Physics that includes classical laws as special cases.

Quantum Physics extends that range to the region of small dimensions.

Just as 'c' the velocity of light signifies universal constant, the Planck's constant characterizes Quantum Physics.

$$h = 6.65 \times 10^{-27}\, erg.\sec$$

$$h = 6.625 \times 10^{-34}\, Joule.\sec$$

Quantum Mechanics

It is able to explain

1. Photo electric effect
2. Black body radiation
3. Compton effect
4. Emission of line spectra

The most outstanding development in modern science was the conception of Quantum Mechanics in 1925. This new approach was highly successful in explaining about the behavior of atoms, molecules and nuclei.

Photo Electric Effect

The emission of electrons from a metal plate when illuminated by light or any other radiation of any wavelength or frequency (suitable) is called photoelectric effect. The emitted electrons are called 'photo electrons'.

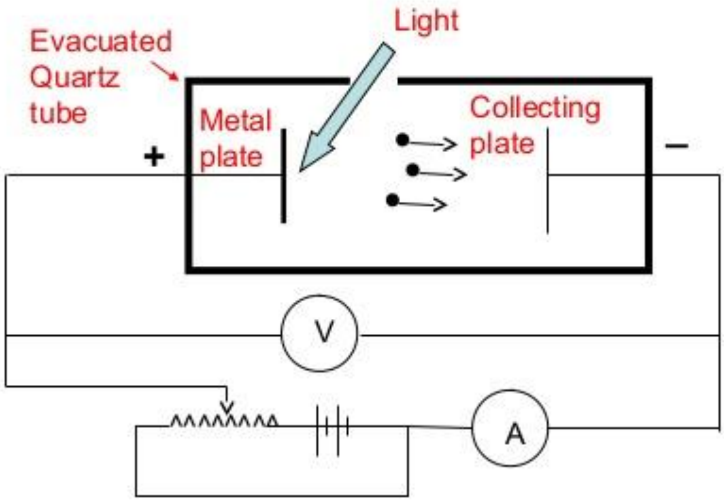


Photo Electric Effect

Experimental findings of the photoelectric effect

1. There is no time lag between the arrival of light at the metal surface and the emission of photoelectrons.
2. When the voltage is increased to a certain value say V_o, the photocurrent reduces to zero.
3. Increase in intensity increase the number of the photoelectrons but the electron energy remains the same.

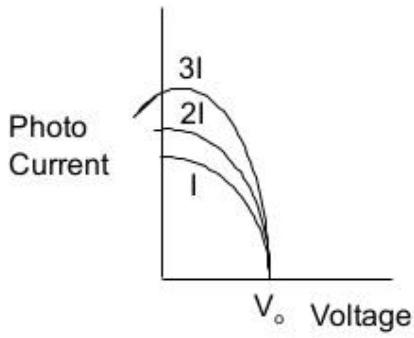
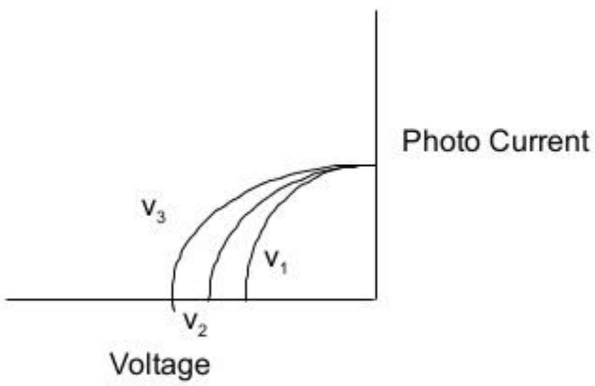


Photo Electric Effect

4. Increase in frequency of light increases the energy of the electrons. At frequencies below a certain critical frequency (characteristics of each particular metal), no electron is emitted.



Einstein's Photo Electric Explanation

The energy of a incident photon is utilized in two ways

1. A part of energy is used to free the electron from the atom known as photoelectric workfunction (W_o).
2. Other part is used in providing kinetic energy to the emitted electron . $\left(\frac{1}{2}mv^2\right)$

$$h\nu = W_o + \frac{1}{2}mv^2$$

This is called Einstein's photoelectric equation.

$$h\nu = W_o + KE_{\max}$$

$$h\nu = h\nu_o + KE_{\max}$$

$$KE_{\max} = h(\nu - \nu_o)$$

If $\nu < \nu_o$, no photoelectric effect

$$W_o = h\nu_o = \frac{hc}{\lambda_o}$$

$$\lambda_o = \frac{hc}{W_o} = \frac{12400}{W_o(eV)}\overset{o}{A}$$

If V_o is the stopping potential, then

$$KE_{\max} = h(\nu - \nu_o)$$

$$eV_o = h\nu - h\nu_o$$

$$V_o = \frac{h\nu}{e} - \frac{h\nu_o}{e}$$

It is in form of $y = mx + c$. The graph with V_o on y-axis and ν on x-axis will be a straight line with slope h/e

Photons

Einstein postulated the existence of a particle called a photon, to explain detailed results of photoelectric experiment.

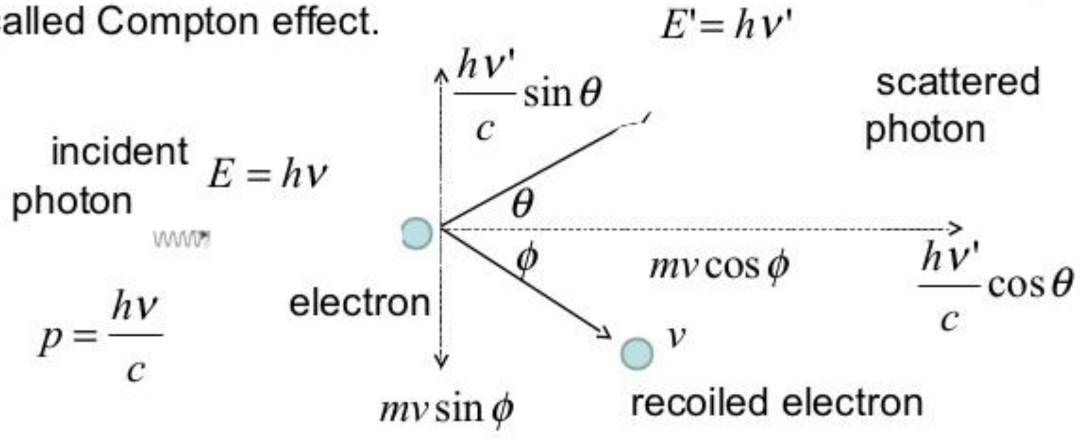
$$E_p = h\nu = \frac{hc}{\lambda}$$

Photon has zero rest mass, travels at speed of light

Explains "instantaneous" emission of electrons in photoelectric effect, frequency dependence.

Compton Effect

When a monochromatic beam of X-rays is scattered from a material then both the wavelength of primary radiation (unmodified radiation) and the radiation of higher wavelength (modified radiation) are found to be present in the scattered radiation. Presence of modified radiation in scattered X-rays is called Compton effect.



From Theory of Relativity, total energy of the recoiled electron with v ~ c is

$$E = mc^2 = K + m_o c^2$$

$$K = mc^2 - m_o c^2$$

$$K = \frac{m_o c^2}{\sqrt{1 - v^2/c^2}} - m_o c^2$$

$$K = m_o c^2 \left[\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right]$$

Similarly, momentum of recoiled electron is

$$mv = \frac{m_o v}{\sqrt{1 - v^2/c^2}}$$

Now from Energy Conversation

$$h\nu = h\nu' + m_o c^2 \left[\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right] \qquad \text{(i)}$$

From Momentum Conversation

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + \frac{m_o v}{\sqrt{1 - v^2/c^2}}\cos\phi \qquad \text{(ii)} \quad \text{along x-axis}$$

and

$$0 = \frac{h\nu'}{c}\sin\theta - \frac{m_o v}{\sqrt{1 - v^2/c^2}}\sin\phi \qquad \text{(iii)} \quad \text{along y-axis}$$

Rearranging (ii) and squaring both sides

$$\left(\frac{h\nu}{c}-\frac{h\nu'}{c}\cos\theta\right)^2=\frac{m_o^2v^2}{1-v^2/c^2}\cos^2\phi \qquad \text{(iv)}$$

Rearranging (iii) and squaring both sides

$$\left(\frac{h\nu'}{c}\sin\theta\right)^2=\frac{m_o^2v^2}{1-v^2/c^2}\sin^2\phi \qquad \text{(v)}$$

Adding (iv) and (v)

$$\left(\frac{h\nu}{c}\right)^2+\left(\frac{h\nu'}{c}\right)^2-\frac{2h^2\nu\nu'}{c^2}\cos\theta=\frac{m_o^2v^2}{1-v^2/c^2} \qquad \text{(vi)}$$

From equation (i)

$$\frac{h\nu}{c}-\frac{h\nu'}{c}+m_oc=\frac{m_oc}{\sqrt{1-v^2/c^2}}$$

On squaring, we get

$$\left(\frac{h\nu}{c}\right)^2+\left(\frac{h\nu'}{c}\right)^2+m_o^2c^2-\frac{2h^2\nu\nu'}{c^2}+2hm_o(\nu-\nu')=\frac{m_o^2c^2}{1-v^2/c^2}$$

(vii)

Subtracting (vi) from (vii)

$$-\frac{2h^2\nu\nu'}{c^2}(1-\cos\theta)+2hm_o(\nu-\nu')=0$$

$$2hm_o(\nu-\nu')=\frac{2h^2\nu\nu'}{c^2}(1-\cos\theta)$$

$$m_o(\nu-\nu')=\frac{h\nu\nu'}{c^2}(1-\cos\theta)$$

But $\nu = \frac{c}{\lambda}$ and $\nu' = \frac{c}{\lambda'}$ So,

$$m_o c\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \frac{h}{\lambda\lambda'}(1 - \cos\theta)$$

$$m_o c\left(\frac{\lambda' - \lambda}{\lambda\lambda'}\right) = \frac{h}{\lambda\lambda'}(1 - \cos\theta)$$

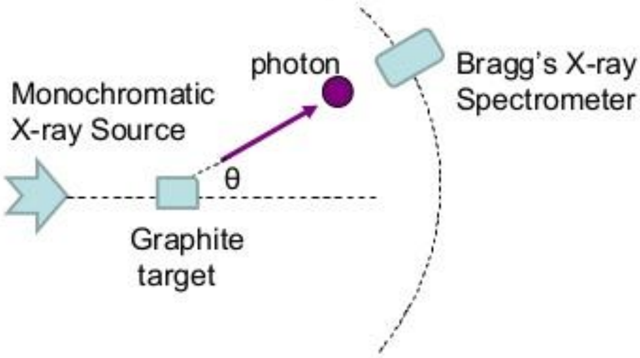
$$\lambda' - \lambda = \Delta\lambda = \frac{h}{m_o c}(1 - \cos\theta)$$

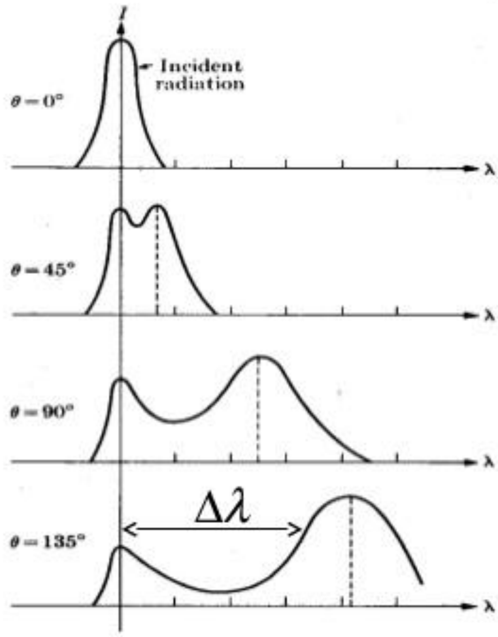
$\Delta\lambda$ is the Compton Shift.

It neither depends on the incident wavelength nor on the scattering material. It only on the scattering angle i.e. θ

$\frac{h}{m_o c}$ is called the Compton wavelength of the electron and its value is 0.0243 Å.

Experimental Verification





1. One peak is found at same position. This is unmodified radiation
2. Other peak is found at higher wavelength. This is modified signal of low energy.
3. $\Delta\lambda$ increases with increase in θ.

Compton effect can't observed in Visible Light

$$\Delta\lambda = \frac{h}{m_o c}(1-\cos\theta) = 0.0243\,(1-\cos\theta)\ \text{Å}$$

$\Delta\lambda$ is maximum when (1- cosθ) is maximum i.e. 2.

$$\Delta\lambda_{max} = 0.05\ \text{Å}$$

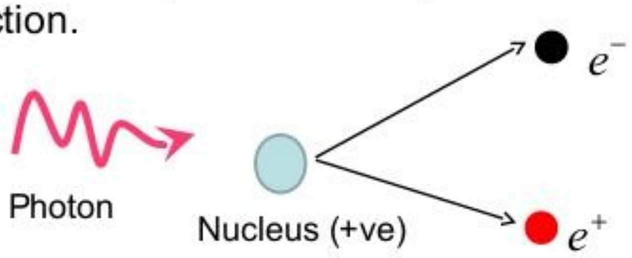
So Compton effect can be observed only for radiation having wavelength of few Å.

For $\lambda =$ 1Å $\quad\Delta\lambda$ ~ 1%

For $\lambda =$ 5000Å $\quad\Delta\lambda$ ~ 0.001% (undetectable)

Pair Production

When a photon (electromagnetic energy) of sufficient energy passes near the field of nucleus, it materializes into an electron and positron. This phenomenon is known as pair production.



In this process charge, energy and momentum remains conserved prior and after the production of pair.

The rest mass energy of an electron or positron is 0.51 MeV (according to $E = mc^2$).

The minimum energy required for pair production is 1.02 MeV.

Any additional photon energy becomes the kinetic energy of the electron and positron.

The corresponding maximum photon wavelength is 1.2 pm. Electromagnetic waves with such wavelengths are called gamma rays (γ).

Pair Annihilation

When an electron and positron interact with each other due to their opposite charge, both the particle can annihilate converting their mass into electromagnetic energy in the form of two γ - rays photon.

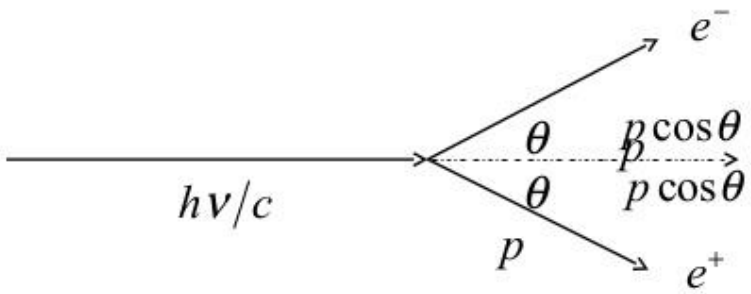
$$e^- + e^+ \rightarrow \gamma + \gamma$$

Charge, energy and momentum are again conversed. Two γ - photons are produced (each of energy 0.51 MeV plus half the K.E. of the particles) to conserve the momentum.

Pair production cannot occur in empty space

From conservation of energy $$h\nu = 2m_o c^2 \gamma$$

here m_o is the rest mass and $\gamma = 1\big/\sqrt{1 - v^2/c^2}$



In the direction of motion of the photon, the momentum is conserved if

$$\frac{h\nu}{c} = 2p\cos\theta$$

$$h\nu = 2cp\cos\theta \qquad \text{(i)}$$

Momentum of electron and positron is

$$p = m_o v\gamma$$

Equation (i) now becomes

$$h\nu = 2m_o cv\gamma\cos\theta$$

$$h\nu = 2m_o c^2\gamma\left(\frac{v}{c}\right)\cos\theta$$

But $\quad v/c < 1 \quad$ and $\quad \cos\theta \le 1$

$$h\nu < 2m_o c^2\gamma$$

But conservation of energy requires that

$$h\nu = 2m_o c^2\gamma$$

Hence it is impossible for pair production to conserve both the energy and momentum unless some other object is involved in the process to carry away part of the initial photon momentum. Therefore pair production cannot occur in empty space.